Automorphisms In Symmetric Balanced Incomplete Block Designs: A Comprehensive Analysis

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Abstract

This study presents a permutation α on a set X using the disjoint cycle representation, with each cycle in the representation on X. This paper considers a symmetric balanced incomplete block design (v, k, λ) as a B-cyclic design with an automorphism α that permutes its objects and blocks in a cycle of length v. The results indicate that a cyclic automorphism α and a set of objects in a single block completely determine the entire experimental design. The residues modulo v corresponding to the automorphism of the cyclic design: $i \to i+1$ *and* $B_i \to B_{i+1}$*, as being the objects and block subscripts respectively have equally been discussed with their properties stated and proved.*

Keywords: Automorphism, Symmetric Balanced Incomplete Block Design and Cyclic Automorphic Symmetric Balanced Incomplete Block Design

Introduction

The theory of design of experiments came into being largely through the work of R. A. Fisher and F. Yates [2] in the early 1930s. They were motivated by questions of design of careful field experiments in agriculture. The applicability of experimental design theory is now very widespread. Block designs are used in experimental planning with the purpose of maximizing the information extracted from a given number of experiments. An arrangement of v treatments in b blocks, each of size k, where k<v, is called balanced incomplete block (BIB) design if:

- Each treatment appears exactly r times where r is defined by $r = {}^{bk}/_{v}$……. *Equation 1*
- Each treatment occurs no more than once per block,
- Each unordered pair of its treatment appears in exactly λ blocks where λ is defined by; $\lambda = r(k-1)(v-1)$ ………..*Equation 2*

These conditions are necessary but not sufficient for the existence of designs.
There are three main concepts of balancing in incomplete block designs, namely

- Variance Balanced,
- Efficiency Balanced,
- Neighbour Balanced.

An incomplete block design is said to be proper if k < v, i.e $k_1 = k_2 = \cdots \ldots = k_b = K$ and for a proper balanced incomplete block design, Fisher's inequality b ≥ v, or equivalently r ≥ k, holds. A balanced incomplete block design is said to be symmetric if v = b, and consequently r=k from bk=rv. It can be shown that for BIBDs any two blocks have λ treatments in common. A balanced incomplete block design is said to be unreduced if it is obtained by taking all combinations of v treatments k at a time. A balanced incomplete block design is said to be resolvable if the set of blocks can be partitioned into subsets, such that the blocks in any subset contain every treatment exactly once. A balanced incomplete block design is called α-Resolvable if its blocks can be divided into r groups, each consisting of b blocks such that in each group every treatment appears exactly α times.

Automorphism

With respect to experimental design, a function is considered an automorphism if f(a,b) = f(a).f(b), ∀a,b∈G. It is helpful to notice that the identity function: f(x) = x, is the trivial isomorphism from a group to itself. The simplest and easiest way to imagine non-trivial automorphism is to think of a cyclic group and literally create a reverse mapping to itself. So, given G = <S.> = {a0, a1, ..., an}, ∋, G is cyclic group, assume that the cycle is of the form: a0→a1→...→an→a0. Then, one may see that a permutation or a bisection of the symmetry that causes its structure to remain unchanged is the mapping f (ai) = an−i. When this mapping is performed, then the adjacency operation is preserved. Arguably using the cyclic group functions, a half circle degree Symmetrical permutation is an automorphism. So,

$f(a_{i,}\, a_{i+1}) = (f(a_{2i+1})) \vee (f(a_{2i+1-n})) = (a_{n-2i-1}) \vee (a_{2(n-i)-1})$
and $f(a_i)f(a_{i+1}) = a_n - i a_n - i - 1 = (a_{2n} - 2i - 1) \vee (a_n - i + n - i - 1 - n) = (a_{2(n-i)-1}) \vee (a_{n-2i-1})$
yields, $(a_{i,}a_{i+1}) = f(a_i)f(a_i + 1), \forall i, i+1 \in [0, n-1]$.

Therefore, this 180-degree Symmetrical permutation is an automorphism for the Cyclic Group *G*. Informally, one is free to think of an automorphism as a symmetry preserving function from a Group to itself.

Isomorphic Symmetric Balanced Incomplete-Block Design

Two designs (X, A) and (Y, B) are two designs with $|X|=|Y|$. (X, A) and (Y, B) are isomorphic if there exists a bijection $\alpha: X \to Y$ such that: $[\alpha(x): x \in A] = B$.

Statistically speaking, renaming every point $x \in X$ by $\alpha(x)$, the collection of blocks A is transformed into B and the bijection α is called an isomorphism.

Definition 1: Automorphic symmetric Balanced Incomplete-block Design:

Let (P; B) be a design with parameters (v,k,λ) and g a permutation on P. Then g induces a permutation on the k-subsets of P by the natural action (x1, ..., xk) → (g(x1), ...g(xk)). If g also induces a permutation on B we call g an automorphism of the design (P, B). Specifically, a group of permutations on the points of a BIB design that preserves its blocks is called an automorphism group of that BIB design.

Definition 2: Resolvable design:

A block design of b blocks in which each of v treatments is replicated r times is said to be resolvable if b blocks can be divided into r sets of $\frac{b}{r}$ blocks each, such that every treatment appears in each set precisely once and a resolvable design, b must be a multiple of r.

Definition 3: Dual Design:

The dual of a BIB design with parameters (v, b, r, k, λ); is obtained by interchanging the treatments and blocks in the original design. The dual of a BIB design is not always a BIB design, however, for a SBIBD the dual is also SBIBD with the same parameters.

Definition 4: Residual Design

In a symmetric BIB design with parameters $(v = b, r = k, \lambda)$ deleting one block and also those treatments which appear in the deleted block from the remaining (b-1) blocks, leads to a residual design. The residual design is also a BIB design with parameters $(v^* = v - k, b^* = b - 1, r^* = r, k^* = k - \lambda, \lambda^* = \lambda)$

Definition 5: Derived Design:

By deleting any block of a symmetric BIB design with parameters $(v = b, r = k, \lambda)$ and retaining all the treatments in $(b - 1)$ blocks that appear in the deleted blocks. A derived design is obtained. The parameters of the derived BIB design are

$(V^*=k;\ b^*= (b-1);\ r^*= (r-1);\ k^* = \lambda;\ \lambda^*= (\lambda-1))$

Motivation

The generation of block designs is an unresolved problem in combinatorial mathematics. Some methods of construction of Balanced Incomplete Block Designs (BIBDs) have been suggested, these include the trade-off method, difference method, variety cutting and construction from finite permutation groups among others. However, the construction techniques have not been exhausted because there are many parameters sets for which the existence of balanced incomplete block designs have not been determined. Due to many open questions and conjectures about existence of BIBDs, this paper presents in detail the properties and construction of cyclic automorphism of symmetric balanced incomplete block designs

Literature Review

The theory of design of experiments came into being largely through the work of R. A. Fisher and F. Yates in the early 1930s. They were motivated by questions of design of careful field experiments in agriculture, but the applicability of this theory is now very widespread.

According to Osuolale and Oteunrim [7], a symmetric balanced incomplete block design is a type of design where b = v and r = k and every two distinct blocks have λ points that are common. Also, apart from other required conditions r − λ should be a perfect square if and only if v is even. They constructed symmetric BIBDs using residual designs in which they recommended further research on the necessary and sufficient conditions.

According to Jen et al [4], in their study which they investigated properties of construction of balanced incomplete block design to bridge the gap of knowledge from the previous studies, they found out that the models are replicated for binary designs say v variables in b blocks of k plots each. However, their model has been criticized as not being held for triple designs, that is when v, b and k are examined together.

Suden and Mogan [9], investigated several methods of construction of symmetric balanced incomplete block designs by using group divisible design in two classes of block size three. The findings of this investigation showed that all the constructed SBIBD designs achieved a lower bound of their matrix elements. A good number of the SBIBD designs used were found to be optimally established under unique technique of analysis in the infinite field, however the investigation narrowly explained the proof of existences of BIBDs in general.

According to Mathew et al [5], in a study that employed combine independent tests and statistical procedures on BIBD statistical procedures for combined independent test, the study tested hypothesis for common mean vectors of two independent linear models with different variance in a BIBD. This method of testing different hypothesis was suitable for some conditions of the study. The conditions were equality of the treatment effects and testing the significance of the treatment variance parameter in balanced incomplete block designs. The findings revealed a significance in the BIBDS with the bigger samples. However, for smaller samples there was no significance with the problem of existence of designs remaining un-addressed.

Gokpinar et al [3], employed non parametric test where data sets ranked with ties under balanced incomplete block design, were used to show that parameters in BIBD should satisfy some condition causing restriction, that is to say, if λ (t − 1) = r(k − 1) and b = t then BIBD is said to be symmetrical, otherwise asymmetrical. They concluded that the power and size, of the tests using BIBDs are limited to these conditions.

Debashis and Lakshmi [1], researched on properties of quasi- symmetric 2-design and showed the block designs have two intersection points x and y with x greater than zero and y greater than x. the study explored the construction for quasi-2-design with block intersection numbers Q and Q+1 where Q is a prime number under conditions of cardinality. The study revealed that it is possible to construct a family of proper quasi-symmetric design with intersections numbers Q and Q+1, where Q is any prime number. The study observed that when the inner design is balanced with respect to blocks of a symmetric incomplete bock design, then the SBIBD must have a quasi-3 feature.

In a study conducted by Nilson [6] proof that a residual design of a related symmetric balanced incomplete block designs is balanced if it is a quasi-symmetry was shown. The study also highlighted that sufficient and necessary conditions must be considered for the numbers that intersect. For the existence properties to hold, intersecting rows and columns in a BIBD must be properly formed.

Yasmin et al [11] in their study on Construction of Balanced Incomplete Block Designs Using Cyclic Shifts argued out that balanced incomplete block designs (BIBDs) play important role in design of experiments since they ensure that treatments are compared with equal precision. The study presented a method to construct BIBDs of some infinite series by using cyclic shifts. The construction method expressed some standard properties of BIBD design through examining the sets of shifts. The study showed that there is no need to construct the blocks of the actual design to obtain the properties of the designs, instead stressed the determination of the off-diagonal elements of the concurrence matrix from the set of the shifts. The study further compiled a catalogue of BIBDs for $3 \leq v \leq 8$ with $k = v - 1$, $19 \leq v \leq 100$ with $k = 4$ and 5 restrictions with $r < 60$.

Methodology

Given two designs on an equal point set which are symmetric balanced incomplete block designs, the sum construction involves generating a collection of all the blocks in both of the arguably symmetric balanced incomplete block designs used. This study employs the idea of sum construction. Two block designs B and B'. are said to be isomorphic if there is a one-to-one mapping α of objects and blocks of design B onto those of B' such that if x is an object space and B_j is a block design of B that is to say,

$\alpha: x_i \rightarrow x'_i = \alpha(x_i)$An object of B'
$\alpha: B_j \rightarrow B'_j = \alpha\left(B_j\right)$ A block of B'
Then

$$x_i \in B_j \text{ if and only if } \alpha(x_i) \in \alpha(B_i)$$

Basing our argument on the above concept, this study has formed the automorphisms of design B by a method known as sum construction. Further, two block designs are isomorphic if they have the same incidence relationship. If incidence B = incidence B' then the mapping α is called automorphism of B.

The methodology employed in this study involved the formation of automorphism of any block design, say B, through formation of cyclic groups.

The essence here is if $\alpha_1$and α_2 are two automorphism of B, then the product of mapping $\alpha_1\alpha_2$ is also an automorphism and their inverse mappings $\alpha_1^{-1}$and α_2^{-1} are automorphisms too.

A method proposed by Stinson [10] was then used to construct cyclic automorphisms of symmetric balanced incomplete block designs.

We have further discussed block designs with special kinds of automorphism using relevant examples.

Analysis and Discussions

Automorphism of Balanced Incomplete Block Designs

Suppose *(X, A)* is a BIB design and its automorphism is an isomorphism of this design with itself. Then the bijection α is a permutation of X such that

[α (x): x ∈ A: A ∈ A] = A

The identity mapping on X is always a (trivial) automorphism of the ***BIBD (X, A)*** design with itself. A design may have other nontrivial automorphism.
Examples:
Let *(X, A)* be the following (7, 3, 1) ─BIBD, with the set X containing the points
X treatments= {1, 2, 3, 4, 5, 6, 7} and Blocks(A)= {123, 145, 167, 246, 257, 347, 356}.

Suppose we define the permutation α as follows $\alpha(1)=1$, $\alpha(2)=2$, $\alpha(3)=3$,$\alpha(4)=5$, $\alpha(5)=4$ $\alpha(6)=7$ and $\alpha(7)=6$ and relabel the points in treatments X using the permutated values of α, the blocks of BIBD labelled tA become of the following treatments.

This implies a one-to-one mapping of the symmetric balanced incomplete block design onto itself.
123─123
145─145
167─176
246─257
257─246
347─356
356─347
And therefore, the permutation α is an automorphism of the symmetric balanced incomplete block design. It's often convenient to present a permutation α on a set X using the disjoint cycle representation, with each cycle in the representation on X having the form (x α(x) α(α(x)………..)

For some x ∈ X, eventually we get back to x, creating a cycle. The cycles thus obtained are disjoint and they have lengths that sum to $|X|$. The order of the permutation α is the least common multiple of the lengths of the cycles in the disjoint cycle representation. A fixed point of α is a point X such that α(x) =X; The fixed point of α corresponds to the cycle of lengths one in the disjoint cycle representation of α.

In the example for (7,3,1) - BIBD, the permutation α has disjoint cycle representation (1)(2)(3)(4,5)(6,7) which is a permutation of some order with 3 fixed points

A set of all automorphisms of a symmetric balanced incomplete block design (X, A) forms automorphism group under the operation of composition of permutations. The automorphism group is a subgroup of the symmetric group $S_{|X|}$ where S_v is the group consisting of all v! Permutations on a set of v elements. In other words, automorphism groups of designs are very good examples of permutations groups (subgroups of S_v)

For example, the balanced incomplete block design (7,3,1) used as design (X, A) in the previous example has another automorphism β= (1,2,4,3,6,7,5). With the composition $\gamma = \alpha\,\beta$ is defined as $\gamma(x) = \beta(\alpha(x))$ for all x∈X

It can be checked that $\gamma = (1,2,4)(3,6,7)(7)$ thus γ is the automorphism of the balanced incomplete block design because it is a composition of two automorphisms. However, the balanced incomplete block design (X, A) has many other automorphisms.

Cyclic Automorphism Of Balanced Incomplete Block Designs

Given two block designs B and B'. They are said to be isomorphic if there is a one-to-one mapping α of objects and blocks of B onto those of B' such that if x is an object andB_j a block of B that is to say,
$\alpha: x_i \to x'_i = (x_i)\alpha$ An object of B'
$\alpha: B_j \to B'_j = (B_j)\alpha$ A block of B'
Then

$$x_i \in B_j \text{ if and only if } (x_i)\alpha \in (B_i)\alpha$$

Basing our argument on the above concept it is worth noting that two block designs are isomorphic if they have the same incidence relationships and so are essentially the same. If $B' = B$ then the mapping α is called automorphism of B. The automorphism of any block design B always form a group since $\alpha_1$and α_2 are two automorphism of B, the product of mapping $\alpha_1\alpha_2$ is also an automorphism and the inverse mappings $\alpha_1^{-1}$and α_2^{-1} are automorphisms too.

In this paper, we present an example of designs with special kinds of automorphism using the BIBD (13,4,4,1). We consider a block design (13,4,1) with given treatments, the residues 0, 1, 2…, 12, (mod 13) and blocks B_i $i = 0, 1, 2 \ldots, 12\ (mod\ 13)$,the blocks are considered to have the following treatments randomly distributed within the block.
B_0 0, 1, 3, 9
B_1 1, 2, 4, 10
B_2 2, 3, 5, 11
B_3 3, 4, 6, 12
B_4 4, 5, 7, 0
B_5 5, 6, 8, 1
B_6 6, 7, 9, 2
B_7 7, 8, 10, 3
B_8 8, 9, 11, 4
B_9 9, 10, 12, 5
B_{10} 10, 11, 0, 6
B_{11} 11, 12, 1, 7
B_{12} 12, 0, 2, 8
For this case$\alpha: i \to i + 1$, $B_i \to B_{i+1}$ is automorphism of B which permutes the treatments and blocks in each cycle of mode 13.

In general, a symmetric balanced incomplete block design (v, k, λ) is called a B-cyclic if the design has an automorphism α that permutes the objects and also the blocks in a cycle of length v. we note that the cyclic automorphism α and the set of objects in a single block completely determine the entire design as the case shown in the above design.

The blocks B_5 with treatments 5 6 8 1 in the above design has the property that (4.3=12) differences of distinct elements in the blocks yield every difference exactly once except 0 (mod 13) this is illustrated below

$$1 \equiv 6-5$$
$$2 \equiv 8-6$$
$$3 \equiv 8-5$$
$$4 \equiv 5-1$$
$$5 \equiv 6-1$$
$$6 \equiv 1-8$$
$$7 \equiv 8-1$$
$$8 \equiv 1-6$$
$$9 \equiv 1-5$$
$$10 \equiv 5-8$$
$$11 \equiv 6-8$$
$$12 \equiv 5-6$$

Definitions

A set of k residues $D\colon\{a_1, \dots, a_k\}$ modulo V is called a symmetric balanced incomplete block design (v, k, λ)- difference set if for every $difference$ d is not 0 (mod v) and if there exists exactly λ ordered pairs (a_i, a_j), $a_i, a_j \in D$ such that $a_i - a_j \equiv d (\mathrm{mod}\ v)$. Thus, referring to the example used in this paper the block B_5 5,6,8,1 is (13, 4, 1) difference design.

Theorem 1

A set of k residues $D\colon\{a_1, \dots, a_k\}$ modulo v is a symmetric balanced incomplete block design (v, k, λ)-difference set if and only if the sets $B_i\colon\{a_1 + i, a_2 + i, \dots, a_k + i\}$ modulo v, $i = 0, \dots, v-1$ are cyclic (v, k, λ) block designs.

Proof

Suppose the sets $B_i\colon\{a_1 + i, a_2 + i, \dots, a_k + i\}$of residues modulo v, for $i = 0, \dots, v-1$, forms a cyclic block design B, then the cyclic automorphism is $\alpha_i \to i + 1\colon B_i \to B_{i+1,}\ \ i = 0, \dots, v-1$ modulo v. Here if d $\not\equiv 0(\mathrm{mod}\ v)$ the objects 0, d occurs together in exactly λ blocks, that is for λ choices $a_i, a_j \in D$ and t we have,

- *$d = a_j + t$, $0 \equiv a_j + t$ (mod v) with*
- Exactly ordered pairs $((a_i, a_j), a_i a_j \in D)$ with $a_i - a_j \equiv d(mod\ v)$
  Since $t \equiv -a_j$is uniquely determined bya_j,
- D is a difference set

Conversely suppose that$D\colon\{a_1, \dots, a_k\}$ is a (v, k, λ)- difference set then in the set $B_i\colon\{a_1 + i, a_2 + i, \dots, a_k + i\}$of residues of modulo v, for $i = 0, \dots, v-1$, every pair of distinct residues *r,s* occurs together λ times. Let $r - s \equiv d \not\equiv 0(mod\ v)$ then for exactly λ ordered pairs $(a_i, a_j), a_i a_j \in D$ we have

r - s ≡ $a_i - a_j$(mod v)
So both $r - s \equiv a_i + t$ and $s \equiv a_j + t$ belong to B_t, where t is determined by: -
t = r - a_i ≡ s – a_j (mod v).

Thus, the sets B_i are the blocks of a cyclic (v, k, λ) block design B. The residues modulo v corresponding to the automorphism of the cyclic design have, : $i \to i + 1$ and $B_i \to B_{i+1}$, as being the objects and block subscripts respectively.

Bruck R.H (2009) has generalized the idea of a cyclic difference set to that of a group difference set D, which may be used on any finite group G. Let G be a finite group of order V. Suppose that a (v, k, λ) block design B admits G as a regular group of automorphism. By this we mean that if x is a particular object and B_0 is a particular block then as g runs over the elements of $G, (X)g$ and $(B_0)g$ run over the objects and blocks of B. Here we call x the base point if $y = (x)g$, is another object, then $(x)g = (y)g^{-1}$,thus we may identify the objects with the group elements; but a change of base point replaces g by $g^{-1}.g$ for an appropriate g_1. For example: -

If G is an abelian group of order16 generated by a, b, c, d where $a^2 = b^2 = c^2 = d^2 = 1$ then D=$\{a, b, c, d, ab, cd\}$ is a (16, 6, 2)-group difference sets, and if (B_0)g is taken as the set {ag, bg, cg, dg, abg, cdg)} with g running over the 16 elements of G, as a regular group of automorphism. For difference set in groups that may be non-abelian in nature, we suggest the use of the following definition.

Definition

A set of k different elements $D: \{a_1, \dots, a_k\}$ from a group of Gs of orders v I called a (v, k, λ)-group difference set if either of the following conditions holds.

- For every $d \neq 1; d \in G$. There are exactly λ ordered pairs(a_i, a_j), $a_i, a_j \in D$ such that $a_i, a_j^{-1} = d$.
- For every $d \neq 1$ $d \in G$ There existsλordered pairs(a_i, a_j),$a_i, a_j \in D$ such that $a_j^{-1}, a_i = d$

The two conditions above are obviously equal if G is an abelian and also equivalent in every finite group. Now we note from the definition that is immediately described above that k(k-1) = λ(v-1). This is because there are k(k-1) ordered pairs and we are to have λ representations a_i, a_j^{-1} or a_i^{-1},a_j of (v-1) number of elements.

Theorem 2

The properties 1 and 2 of group difference sets are equivalent if B is a balanced incomplete block design with parameters (v, k, λ) admitting the group G of order r as a regular group of automorphism. Given $(x)a_1, \dots (x)a_k$ are the objects of a blockB_0, then $D: \{a_1, \dots, a_k\}$ is a (v, k, λ)-group difference set. Conversely if $D: \{a_1, \dots, a_k\}$ is a (v, k, λ)-group difference set of elements from the group G of order v, then the set $B(g): \{a_1 g, \dots, a_k g\}$ as g runs over G from a (v, k, λ) block design admitting G as a regular automorphism.

Proof

Suppose that B is a (v, k, λ) block design admitting the group G of order v as a regular group of automorphism. If x is any object of B, taking x as the base point, all the objects will be $(x_1)g$, $g \in G$ and we may identify (x)g with g. then if $a_1, \dots, a_k$ are the objects of a block B_0, $(B_0)g$ contains $a_1g, \dots, a_kg$. With $d = 1$ an element of G there are exactly λ blocks pairs (a_i, a_j) such that for some g, $a_ig = d$ and $a_ig = 1$ whenever $a_ia_j^{-1} = d$, but here $g = a_j^{-1}$ is determined by a_j. Hence $D = \{a_{1,\dots,}a_k\}$ is a (v, k, λ) different satisfying property 1

Conversely if $D = \{a_1, \dots, a_k\}$ is a (v, k, λ) different set of elements from G order v satisfying property 1, we construct the sets $B(g): \{a_1g, \dots, a_kg\}$ as g runs over G. If *r; s* are two different elements of G, let $rs^{-1} = d \neq 1$ determine d, and from λ ordered pairs (a_i, a_j) such that $a_ia_j = d$, we determine G from such a pair by $a_1^{-1}r = a_j^{-1}s = g$ whenever both $r = a_ig$ and $s = a_jg$ belong to $B(g)$.

Thus, the sets $B(g)$ are blocks of design B admitting G a regular group of automorphism. We have shown the equivalence of different seta with property 1 to block design B admitting G regularly

For property 2

If $d \neq 1$ then the blocks $a_1, a_2, \dots, a_k$ and $a_1d, a_2d, \dots, a_kd$ have exactly λ elements in common but this says that for exactly λ ordered pairs a_1a_2 we have $a_id = a_j$ whenever $d = a_i^{-1}a_j$.we treat this as a property 2 which is believed to be a consequence of property 1. Conversely if we assume property2, the sets $B(r): \{a_1r, \dots, a_kr\}$ and $B(s): \{a_1s, \dots, a_ks\}$ have exactly λ elements in common whenever they are blocks of a (v, k, λ) design.

Conclusion

In this paper, we conclude that a set of all automorphisms of a symmetric balanced incomplete block design (X, A) forms automorphism group under the operation of composition of the given permutations. The automorphism group is a subgroup of the symmetric group $S_{|X|}$ where S_v is the group consisting of all v! We have equally shown that two block designs are isomorphic if they have the same incidence relationships and so are essentially the same. If $B' = B$ then the mapping α is called automorphism of B. We recommend further research on automorphisms of non-symmetric balanced incomplete block designs.

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Ethics Statement

All methods were performed in accordance with the Technical University of Mombasa (TUM) ethical review guidelines. The study was approved by TUM Research Ethics Committee.

Conflicts of Interest

The authors declare no conflicts of interest.

Author Contributions

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metric BIBDs from Affine